Calculation of nuclear level density within a micro-macroscopic approach

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Many properties of heavy nuclei can be described in terms of the statistical level density. The level density $\rho(E,A)$, where E and A are the nuclear energy and particle number of an isotopically symmetric nucleus, can be calculated by the inverse Laplace transformation of the partition function $\mathcal{Z}(\beta,\alpha)$, where β and α are the Lagrange multipliers, respectively. Within the grand canonical ensemble, the standard Darwin-Fowler saddle point method (SPM) can be applied for the inverse Laplace integration over all variables, including β, which is related to the total energy E, but for large excitation energy U. As usually assumed, the temperature T is related to the well-determined saddle point in the integration over β for a finite Fermi system of a large particle number A. However, many experimental data also exist for low-lying excitation energies U, where such a saddle point does not exist. Therefore, the integral over the Lagrange multiplier β should be carried out [1] more accurately beyond the standard SPM. For another variable, related to the number of nucleons, one can apply the SPM assuming that particle number A is relatively large. In this work, we remove divergence at the critical catastrophe point for zero excitation energy limit of $\rho(E,A)$ and study the shell effects in the level density parameter versus the experimental data by using the periodic orbit theory (POT).

Taking more accurately the inverse Laplace integral over Lagrange multiplier β we have derived analytically the level density $\rho(E,A)$ in terms of the modified Bessel function $I_\nu(S)$ of the entropy S, $\rho \propto S^{-\nu} I_\nu(S)$. Here, $S = 2(aU)^{1/2}$, where U is the excitation energy, $U = E - E_0$, E_0 is the background energy, $E_0 \approx E_{\mathrm{ETF}} + \delta E$, E_{ETF} is the smooth extended Thomas-Fermi (ETF) part of the background energy E_0 [2], and δE is the shell correction energy. The level density parameter a is proportional to the semiclassical POT level density, $g(\lambda)$, with the decomposition in terms of the ETF $g_{\mathrm{ETF}}(\varepsilon)$ [2] and shell corrections $\delta g(\varepsilon)$ components, where both are taken at the chemical potential λ, $\varepsilon = \lambda$, $g(\lambda) \approx g_{\mathrm{ETF}}(\lambda) + \delta g(\lambda)$, for which one has the well-known analytical expressions. The modified Bessel function $I_\nu(S)$ of the order ν is determined by the number of integrals of motion κ, except the energy (for the one particle number integral of motion A, we have $\kappa = 1$). We obtained values of $\nu = 3/2$ for the case of relatively small shell-correction effects ($\nu = \kappa/2 + 1$) and $\nu = 5/2$ for the case of large shell correction effects ($\nu = \kappa/2 + 2$).

Fig. 1 shows the inverse level density parameter $K = A/a$ as function of the particle number A of symmetric nuclear system in the semiclassical POT approximation. The results of these calculations [1] are in a qualitative agreement with the recent experimental data [3], which included in the analysis the excited states in much more nuclei than those known earlier for neutron resonances. For the oscillating shell correction $\delta g(\varepsilon)$, for simplicity, we used the well-known explicitly given POT result for the infinitely deep square well potential because the only single-particle states near the Fermi surface essentially contribute to $\delta g(\varepsilon)$. Its Gaussian averaging with the width $\gamma = 0.3$ has been calculated analytically, which almost identically coincides with the corresponding quantum results of the Strutinsky shell correction method, and it is well describes the major shell structure. The smooth part $g_{\mathrm{ETF}}(\varepsilon)$ is calculated on the basis of the

realistic Skyrme forces SKM* and KDE0v1 with the effective mass [2]. The positions of maxima of the level density parameter a (or oscillating level density $\delta g(\varepsilon)$) cannot be accurately reproduced because of neglecting the spin-orbit interaction. However, according to the POT results for the isomeric state in ^{240}Pu, we took into account the effect of the spin-orbit forces effectively by shifting the curve $K(A)$ in about the period of the major shell structure, $\Delta A = 20$, along the A axis. In spite of very simple explicitly analytical calculations of the inverse level density parameter K, the magnitudes of the periods for the oscillations of $K(A)$ are basically in good agreement with data for particle numbers of the order of 45-150. However, there is a discrepancy for particle numbers in the range of 150-240 for several reasons. Experimental data for K were obtained [3] in good agreement with those for neutron resonances because they are dominating in the specific least mean-square fit using relatively large widths. For this nuclear range the low excitation energies spectra are dramatically different than those of neutron resonances. Another reason might be that the pairing effects should be taken into account along with the shell structure, e.g., in magic nucleus ^{208}Pb. Especially for nuclei in this low excitation energy range, we need the model-independent results of experimental data for level density; see Ref. [1].
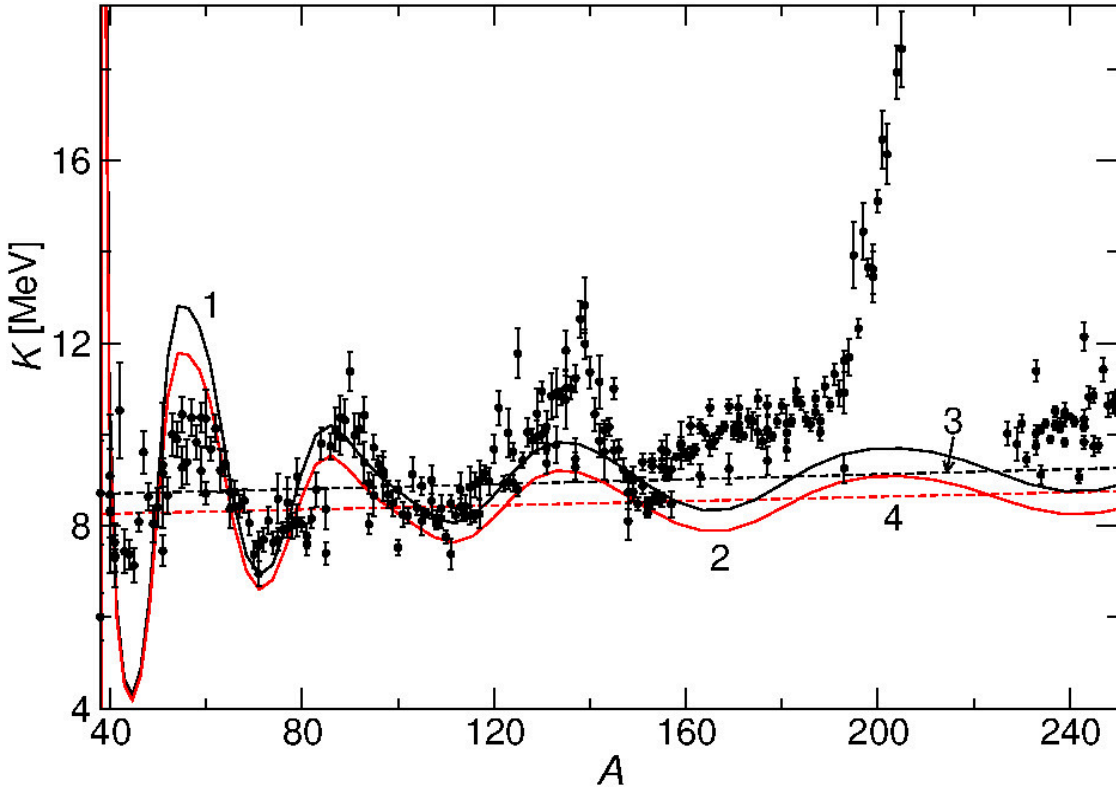

Fig. 1. The inverse level-density parameter $\mathbf{K = A/a}$ (solids "1" for SKM* and "2" for KDE0v1 forces) is shown as function of the particle number $\mathbf{A}$. The smooth part in the ETF approach is taken from Ref. [2] for these two versions of the Skyrme forces SKM* ("3" dashed) and KDE0v1 ("4" dashed). The solid oscillating curves are obtained by using the semiclassical POT approximation for the shell corrections $\boldsymbol{\delta g(\varepsilon)}$ of the single-particle density $\boldsymbol{g(\varepsilon)}$ with the Gauss width averaging parameter $\boldsymbol{\gamma = 0.3}$ in dimensionless energy units $\mathbf{kR}$ where $\mathbf{k = (2m\,\varepsilon/\hbar^2)^{1/2}}$, $\mathbf{R}$ is the radius of a spherical system, $\mathbf{m}$ is the particle mass (see Ref. [1]). The dashed curves "3" and "4" present smooth parts, both include the important effective mass contribution. The chemical potential is $\boldsymbol{\lambda = 40}$ MeV. Experimental values, shown by solid points, are taken from Ref. [3].

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